Multipolar Inner Product Spaces

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Abstract

One may construct n-dimensional metric spaces with p coaxial directions, or poles, that generalize inner product spaces of 2^n orthants to *multipolar inner product spaces* of p^n orthants. Multipolar inner product spaces ($M^n_{G(p)}$) may be defined over a superclass of fields known as *confields* in which a nonassociative operation (*consolidation*) is extensionally equal to addition if $p \leq 2$. Multipolar number systems ($M_{G(p)}$) are instances of such confields. Multipolar number systems may be constructed such that they are commutative unital invertible non-cancellative magmas under consolidation over which multiplication distributes. Nonzero multipolars are Abelian groups under multiplication. One may introduce strict weak orders on p-partite multipolar sets. Multipolar inner product spaces $M^n_{G(p>2)}$ have novel geometric properties (e.g., non-unique pairs of $(n-1)$-sphere antipodes, achirality of objects chiral in Euclidean space, Euler characteristics of 3-orthotopes and their duals, etc.) that may be worthy of further study. Applications may include analysis of data captured via multipolar interval scale items.

1 Introduction

In 1827, Möbius provided a beginning sketch of what has become higher-dimensional vector analysis, writing that an object chiral in a space of dimension n may be mapped to its mirror image via a rotation of π radians through a space of dimension $n+1$.[1] The following document supports the finding that an object chiral in a Euclidean space may become achiral by means other than increase in the space's dimension. Such an object may be embedded in a space of like dimension such that one may map it to its mirror image via translation if that space has > 2 coaxial directions, or poles, per orthogonal axis, each pair of which is separated by an angle of π radians. For such an alternative space–not necessarily higher-dimensional, but multipolar–, computability of vector translation and scalar multiplication depend on how those operations are defined.

To construct multipolar spaces, one may define generalizations of vector spaces (*convector spaces*) equipped with a generalization of an inner product operation (*multipolar inner product*) over number systems that are generalizations of fields (*confields*) such as multipolar number systems ($M_{G(p)}$). Each confield and each convector space is equipped with an addition-like operation (*consolidation*) that may admit non-unique inverses and is accordingly non-associative. (Consolidation may be demonstrated to be extensionally equal to addition if $p \leq 2$.) Multiplication that distributes over consolidation is associative and commutative with identity and inverses. Consequently, multipolar inner product spaces ($M^n_{G(p)}$) that generalize real inner product spaces may be defined.

Multipolar inner product spaces have novel geometric properties. Such spaces may also have applications in the analysis of multipolar interval scale item data as cosine similarity between coordinate vectors of such spaces may be computed.

This document begins with pertinent set-builder notation and definition of operations, then proceeds to presentation of axioms before discussion of geometric properties and directions for further study.

2 Definition of p-partite Multipolar Sets $M_{G(p)}$

For {

$$(x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap}) \in R_{\geq 0} \times R_{\geq 0} \times R_{\geq 0} \times ... \times R_{\geq 0}$$

and

$$(x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp}) \in R_{\geq 0} \times R_{\geq 0} \times R_{\geq 0} \times ... \times R_{\geq 0}$$

,

$$\{x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap}\} = A$$

and

$$\{x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp}\} = B$$

such that $|A_{>0}| \leq 2$ and $|B_{>0}| \leq 2$;

$$x'_{zi} = x_{zi} - x_{z(i+1)} - x_{z(i+2)} - ... - x_{zp}$$

$$x'_{z(i+1)} = x_{z(i+1)} - x_{zi} - x_{z(i+2)} - ... - x_{zp}$$
$$x'_{z(i+2)} = x_{z(i+2)} - x_{zi} - x_{z(i+1)} - ... - x_{zp}$$
$$x'_{z(...)} = x_{z(...)} - x_{zi} - x_{z(i+1)} - ... - x_{zp}$$
$$x'_{zp} = x_{zp} - x_{zi} - x_{z(i+1)} - x_{z(i+2)} - ... - x_{z(p-1)}$$

;

$$x''_{zi} = \text{sgn}(1 + \text{sgn}(x'_{zi}))x'_{zi}$$

},

$$[(x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap})] \equiv [(x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp})]$$
$$\iff$$
$$x''_{ai}, x''_{a(i+1)}, x''_{a(i+2)}, ..., x''_{ap} = x''_{bi}, x''_{b(i+1)}, x''_{b(i+2)}, ..., x''_{bp}$$
$$M_{G(p)} = (R_{\geq 0} \times R_{\geq 0} \times R_{\geq 0} \times ... \times R_{\geq 0})/\equiv$$

3 Order Relations on p-partite Multipolar Sets $M_{G(p)}$

Below are four examples of how one may introduce p unique strict weak order relations on a p-partite multipolar set. A geometric intuition regarding these relations is that each references a unique direction of unbounded magnitude in a space of dimension one, i.e. each relation indicates relative distance from one of p maximal extensions (poles) of a p-partite multipolar number line. Under such an order relation, for each unique absolute value there are $p - 1$ values incomparable under that order relation. An example using a quadripartite multipolar system: sequitive one $(0, 1, 0, 0)$, trinitive one $(0, 0, 1, 0)$, and quadritive one $(0, 0, 0, 1)$ are incomparable with each other under the order relationship *less positive-ward than*, i.e. they are each equidistant from the positive vector of greatest magnitude.

3.1 Example Order Relation: Less Positive-ward Than ($<$Pos)

$$[(x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap})] < \text{Pos}[(x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp})]$$

$$\Longleftrightarrow$$

$x''_{ai} - (\text{sgn}((-(\text{sgn}(x'_{ai}))) + 1)(|(((|(|(|x_{a(i+1)} - x_{a(i+2)}|) - ...|) - x_{ap}|) - x_{ai})|)$
$<$
$x''_{bi} - (\text{sgn}((-(\text{sgn}(x'_{bi}))) + 1)(|(((|(|(|x_{b(i+1)} - x_{b(i+2)}|) - ...|) - x_{bp}|) - x_{bi})|)$

3.2 Example Order Relation: Less Sequitive-ward Than ($<$Seq)

$$[(x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap})] < \text{Seq}[(x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp})]$$

$$\Longleftrightarrow$$

$x''_{ai+1} - (\text{sgn}((-(\text{sgn}(x'_{ai+1}))) + 1)(|(((|(|(|x_{a(i)} - x_{a(i+2)}|) - ...|) - x_{ap}|) - x_{ai+1})|)$
$<$
$x''_{bi+1} - (\text{sgn}((-(\text{sgn}(x'_{bi+1}))) + 1)(|(((|(|(|x_{b(i)} - x_{b(i+2)}|) - ...|) - x_{bp}|) - x_{bi+1})|)$

3.3 Example Order Relation: Less Trinitive-ward Than ($<$Trin)

$$[(x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap})] < \text{Trin}[(x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp})]$$

$$\Longleftrightarrow$$

$x''_{ai+2} - (\text{sgn}((-(\text{sgn}(x'_{ai+2}))) + 1)(|(((|(|(|x_{a(i)} - x_{a(i+1)}|) - ...|) - x_{ap}|) - x_{ai+2})|)$
$<$
$x''_{bi+2} - (\text{sgn}((-(\text{sgn}(x'_{bi+2}))) + 1)(|(((|(|(|x_{b(i)} - x_{b(i+1)}|) - ...|) - x_{bp}|) - x_{bi+2})|)$

3.4 Example Order Relation: Less p-itive-ward Than ($< p$)

$$[(x_{ai}, x_{a(i+1)}, x_{a(i+2)}, ..., x_{ap})] < p[(x_{bi}, x_{b(i+1)}, x_{b(i+2)}, ..., x_{bp})]$$

$$\Longleftrightarrow$$

$x''_{ap} - (\text{sgn}((-(\text{sgn}(x'_{ap}))) + 1)(|(((|(|(|x_{a(i)} - x_{a(i+1)}|) - x_{ai+2}|) - ...|) - x_{ap})|)$
$<$
$x''_{bp} - (\text{sgn}((-(\text{sgn}(x'_{bp}))) + 1)(|(((|(|(|x_{b(i)} - x_{b(i+1)}|) - x_{bi+2}|) - ...|) - x_{bp})|)$

4 p-partite Multipolar Unary Operations

4.0.1 Unary Operation: p-partite Multipolar Absolute Value ($|x|$)

$M_{G(p)} \mapsto M_{G(p)}$
$|(x_{zi}, ..., x_{zp})| = x''_{zi} + x''_{zi+1} + ... + x''_{zp}, 0, ..., 0$

4.0.2 Unary Operation: p-partite Multipolar Sign ($\text{msgn}(x)$)

$M_{G(p)} \mapsto M_{G(p)}$

$$\text{msgn}(x_{zi}, ..., x_{zp}) = \text{sgn}(x''_{zi}), \text{sgn}(x''_{z(i+1)}), ..., \text{sgn}(x''_{zp})$$

5 p-partite Multipolar Binary Operations

5.1 Binary Operation: p-partite Multipolar Consolidation ($\ddot{+}$)

$M_{G(p)} \times M_{G(p)} \mapsto M_{G(p)}$

Consolidation ($\ddot{+}$) may be defined:

$$(x_{ai}, ..., x_{ap}) \ddot{+} (x_{bi}, ..., x_{bp}) = (x''_{ai} + x''_{bi}), (x''_{ai+1} + x''_{bi+1}), ..., (x''_{ap} + x''_{bp})$$

Consolidation may accommodate non-unique pairs of inverse elements and subsequently may not exhibit properties consistent with associativity such as outputs bound by the cancellation law. Extensional equality between addition and consolidation may be shown with $M_{Z(2)}$.

5.2 Binary Operation: p-partite Multipolar Multiplication ($\bullet$)

$M_{G(p)} \times M_{G(p)} \mapsto M_{G(p)}$

Suppose $\{a, b, c, d\} \in M_{G(p)}$;

$$\text{msgn}(a) \bullet \text{msgn}(b) = \text{msgn}(c) = \text{msgn}(a) \bullet \text{msgn}(d) \iff \text{msgn}(b) = \text{msgn}(d)$$

i.e., in a given system $M_{G(p)}$, a given product sign and a given factor sign imply one and only one sign for the other multiplied factor.

The p-partite multipolars are Abelian groups under multiplication that distributes over consolidation. Sign multiplication tables for multipolar systems may be isomorphic to Latin squares such as the Cayley tables of finite Abelian groups. One may construct as many multipolar systems of a given p as there are finite Abelian groups of order p; different systems of the same p (sign cardinality) may be distinguished by the finite Abelian group used to construct the multiplication operation for the system.

Multiplication ($\bullet$) for tripartite multipolars ($M_{Z(3)}$): $(x_{a1}, x_{a2}, x_{a3}) \bullet (x_{b1}, x_{b2}, x_{b3}) =$
$((x''_{a1} \times x''_{b1}) + (x''_{a2} \times x''_{b3}) + (x''_{a3} \times x''_{b2})),$
$((x''_{a1} \times x''_{b2}) + (x''_{a2} \times x''_{b1}) + (x''_{a3} \times x''_{b3})),$
$((x''_{a1} \times x''_{b3}) + (x''_{a2} \times x''_{b2}) + (x''_{a3} \times x''_{b1}))$

Multiplication ($\bullet$) for quadripartite multipolars ($M_{Z(4)}$): $(x_{a1}, x_{a2}, x_{a3}, x_{a4}) \bullet (x_{b1}, x_{b2}, x_{b3}, x_{b4}) =$
$((x''_{a1} \times x''_{b1}) + (x''_{a2} \times x''_{b4}) + (x''_{a3} \times x''_{b3}) + (x''_{a4} \times x''_{b2})),$
$((x''_{a1} \times x''_{b2}) + (x''_{a2} \times x''_{b1}) + (x''_{a3} \times x''_{b4}) + (x''_{a4} \times x''_{b3})),$
$((x''_{a1} \times x''_{b3}) + (x''_{a2} \times x''_{b2}) + (x''_{a3} \times x''_{b1}) + (x''_{a4} \times x''_{b4})),$
$((x''_{a1} \times x''_{b4}) + (x''_{a2} \times x''_{b3}) + (x''_{a3} \times x''_{b2}) + (x''_{a4} \times x''_{b1}))$

Multiplication ($\bullet$) for quadripartite multipolars ($M_{K(4)}$): $(x_{a1}, x_{a2}, x_{a3}, x_{a4}) \bullet (x_{b1}, x_{b2}, x_{b3}, x_{b4}) =$
$((x''_{a1} \times x''_{b1}) + (x''_{a2} \times x''_{b2}) + (x''_{a3} \times x''_{b3}) + (x''_{a4} \times x''_{b4})),$
$((x''_{a1} \times x''_{b2}) + (x''_{a2} \times x''_{b1}) + (x''_{a3} \times x''_{b4}) + (x''_{a4} \times x''_{b3})),$
$((x''_{a1} \times x''_{b3}) + (x''_{a2} \times x''_{b4}) + (x''_{a3} \times x''_{b1}) + (x''_{a4} \times x''_{b2})),$
$((x''_{a1} \times x''_{b4}) + (x''_{a2} \times x''_{b3}) + (x''_{a3} \times x''_{b2}) + (x''_{a4} \times x''_{b1}))$

6 Constructing Convector Spaces

6.1 Vector Equivalence

Vectors may be expressed as columns with their equality defined as below. Suppose that $u, v \in M^n_{G(p)}$. Then $u = v$ if $[u]_i = [v]_i$ and $1 \leq i \leq n$.

6.2 Binary Operation: Vector Consolidation

$V \times V \mapsto V$

Vectors expressed as columns may be consolidated supposing that $u, v \in M^n_{G(p)}$ and $1 \leq i \leq n$. The consolid of u and v is the vector $u \ddot{+} v$ as indicated: $[u \ddot{+} v]_i = [u]_i \ddot{+} [v]_i$

6.3 Binary Operation: Scalar Multiplication

$M_{G(p)} \times V \mapsto V$

Vectors expressed as columns may be multiplied by scalars. Supposing $u \in M^n_{G(p)}$ and $\alpha \in M_{G(p)}$, then the scalar multiple of u by α is the vector αu defined by $[\alpha u]_i = \alpha[u]_i$ and $1 \leq i \leq n$.

6.3.1 Binary Function: Multipolar-to-Real Product ($\mathrm{mrp}(x, y)$)

$M_{G(p)} \times M_{G(p)} \mapsto R$

Supposing $\{x, y\} \in M_{G(p)}$,

$$\mathrm{mrp}(x,y) = \begin{cases} |x||y| \iff & |\,\mathrm{msgn}(\mathrm{msgn}(x) \ddot{+}\, \mathrm{msgn}(y))| = 1 \\ -(|x||y|) \iff & |\,\mathrm{msgn}(\mathrm{msgn}(x) \ddot{+}\, \mathrm{msgn}(y))| = 0 \end{cases}$$

6.3.2 Binary Operation: Multipolar Inner Product ($u \boxdot v$)

$V \times V \mapsto R$

Supposing $\{(u_i, u_{i+1}, ..., u_n), (v_i, v_{i+1}, ..., v_n)\} \in M^n_{G(p)}$, multipolar inner product of two vectors $u = (u_i, u_{(i+1)}, ..., u_n)$ and $v = (v_i, v_{(i+1)}, ..., v_n)$ is defined as:

$$u \boxdot v = \sum_{i=1}^{n} \mathrm{mrp}(u_i v_i) = \mathrm{mrp}(u_1 v_1) + \mathrm{mrp}(u_2 v_2) + ... + \mathrm{mrp}(u_n v_n)$$

7 Algebraic Structures

7.1 Confields: Relaxing Addition's Associative Property

Algebraic structures with addition tend to be defined such that addition is associative. Noteworthy exceptions to this include neofields[4], left neofields[3], as well as the nonassociative number theory of Evans[2]. The addition operation of some neofields may neither commute nor associate and in those ways may differ from the addition of ordinary arithmetic or vector addition. In the case of Evans' system, Williams found that its properties include addition that neither commutes nor associates and has "no analogs in ordinary arithmetic."[5]

If one assumes the field axioms and suspends the associativity of addition, a given element may not be bound to have a unique additive inverse and subsequently the cancellation law may not hold for outputs of such an operation. An expression of such axioms as *confield* axioms appears below. Number systems that are confields such as p-partite multipolars $M_{G(p)}$ (defined above) may be used to construct generalizations of inner product spaces known as *multipolar inner product spaces* (defined below) that qualify as metric spaces.

Let a confield M be a set with two operations: consolidation and multiplication. The result of consolidating a and b is called the "consolid" or "c-sum" of a and b and may be denoted $a + b$ or $a \ddot{+} b$. The result of multiplying a and b is called the "product" of a and b, and may be denoted ab or $a \bullet b$. These operations are required to satisfy seven properties, referred to as "confield axioms." In these axioms, a, b, c, and d are arbitrary elements of the confield M.

1. Associativity of multiplication: $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
2. Commutativity of consolidation and multiplication: $a \ddot{+} b = b \ddot{+} a$ and $a \bullet b = b \bullet a$
3. Consolidative identity: an element exists in M, 0, such that $a \ddot{+} 0 = a$.
4. Multiplicative identity: an element exists in M, 1, such that $a \bullet 1 = a$.
5. Consolidative inverses: For every $a \in M$ there exists one or more elements known as a consolidative inverse $b \in M$ such that $a \ddot{+} b = 0$.
6. Multiplicative inverses: for every $a \neq 0 \in M$, there exists an element in M that may be denoted by a^{-1} and called "the multiplicative inverse" of a, such that $a \bullet a^{-1} = 1$.
7. Left and right distributivity of multiplication over consolidation:

$$a \bullet (b \ddot{+} c) = (a \bullet b) \ddot{+} (a \bullet c) = (b \ddot{+} c) \bullet a$$

One may construct a class of confield number systems (p-partite multipolars) such that a given nonzero number may have $p-1$ consolidative inverses. While consolidation is not bound by axiom to associate, it may. As $M_{Z(2)}$ is isomorphic to R it may exhibit extensional equality between confield consolidation and field addition.

7.2 Convector Spaces: Nonassociative Vector Consolidation

Axioms regarding vector addition that define vector spaces may be suspended and replaced with those pertaining to vector consolidation, resulting in *convector spaces*. Multiplication by a confield scalar may subsequently distribute over such consolidation. One may define vector consolidation for a convector space over a confield such that vector spaces over fields are a subset of convector spaces over confields, as is done below.

A convector space over a confield M may be defined as a set V with two operations, vector consolidation and scalar multiplication. The result of consolidating any two vectors v and w is called the "consolid" or "c-sum" of v and w and may be denoted $v \ddot{+} w$. The result of scalar multiplication of confield element a and vector v is called the "product" of a and v, and may be denoted av or $a \bullet v$. These operations are required to satisfy seven axioms of convector spaces. In these axioms, u, v, and w are arbitrary vectors and a, b are scalars of the confield M.

1. Commutativity of consolidation: $u \ddot{+} v = v \ddot{+} u$
2. Identity element of consolidation: There exists an element $0 \in V$, called the zero vector, such that $v \ddot{+} 0 = v$ for all $v \in V$.
3. Inverse elements of consolidation: For every vector $v \in V$, there exists one or more elements $u \in V$ called a consolidative inverse of v such that $v \ddot{+} u = 0$.
4. Compatibility of scalar multiplication with confield multiplication: $a(bv) = (ab)v$
5. Identity element of scalar multiplication: $1v = v$, where 1 denotes the multiplicative identity in M.
6. Distributivity of scalar multiplication with respect to vector consolidation: $a(u \ddot{+} v) = au \ddot{+} av$
7. Distributivity of scalar multiplication with respect to confield consolidation: $(a \ddot{+} b)v = av \ddot{+} bv$

7.3 Multipolar Inner Product Spaces

7.3.1 Algebraic Properties

Proper inner product spaces may be characterized by three properties. Suspending the necessity of linearity in the first or second argument permits the generalization of inner product spaces to *multipolar inner product spaces*. A multipolar inner product space is a convector space V over a confield M with a multipolar inner product operation (such as $\boxdot$ defined above) that maps two vectors to a scalar in M such that it is conjugate symmetric and positive-definite for a nonzero vector with itself. With the length of a vector defined as equal to the square root of its multipolar inner product with itself ($\|v\| = \sqrt{v \boxdot v}$), one may calculate the angle between two vectors using the equation $\angle(u,v) = \arccos \frac{u \boxdot v}{\|u\|\|v\|}$ as they might for an inner product space proper. One may use this computation in applications such as cosine similarity analysis.

7.3.2 Geometric Properties of $M^n_{G(p)}$ and Subjects for Further Research

On a given axis $M^1_{G(p)}$, there are p coaxial directions from the origin, or poles. Each pair of the p directions is π radians apart. One may visualize a multipolar axis as a complete graph (mystic rose) with p nodes such that each node indicates the unique direction of a coordinate vector of unbounded magnitude and the midpoint of each graph edge indicates the location of the origin. In such a visualization, mapping the same number to more than one representation may permit one to intuitively apprehend that the angle between any two points of the multipolar continuum is π radians. By computation of distances, no two coordinate vectors of unlike sign may have a coordinate vector of a third sign between them.

One may multiply a coordinate vector on a p-partite multipolar axis with one of $p-1$ nonpositive scalar units such that its product is mapped to one of its $p-1$ coaxial consolidative inverse vectors π radians away. For reasons discussed below, such consolidative inverse vectors may be referred to as *linear antipodes*. Any member of a set of linear antipodes may be transformed to any of the others in that set by such nonpositive unit scalar multiplication.

In a multipolar space $M^n_{G(p)}$, each of the n axes intersect at the origin and form right angles ($\pi/2$ radians) with the other $n-1$ axes. The quantity of orthants in a multipolar space of dimension n equals p^n. Linear functions expressed via point-slope or parametric form have p-poled outputs such that each pole is π radians apart from the other.

One may embed in any of the orthants of $M^n_{G(p)}$ an object congruent to any in R^n. If such an embedded object is chiral in R^n and is contained by an orthant of $M^n_{G(p>2)}$, it may be shown to be achiral in $M^n_{G(p>2)}$ as per translation across an axis.

In multipolar planes $M^2_{G(p>2)}$, unit circles centered at the origin do not necessarily constitute Hamiltonian cycles among the p^2 orthants of each plane. If a unit circle constitutes such a cycle, it is not unique to its plane, i.e., there are multiple orthant sequences around those unit circles (e.g., $M^2_{K(4)}$).

In an orthant of a multipolar space, a given point Q on a unit circle or $(n-1)$-sphere may have an origin-centered linear function that passes through it. Such a linear function may intersect the unit circle or $(n-1)$-sphere at $p-1$ other points π radians away from Q, Q's linear antipodes. The total quantity of antipodes of Q, or points π radians away from Q on the surface of a unit circle or $(n-1)$-sphere, equals $(p-1)^n$. The cardinality of the subset of those points that do not lie on the linear function passing through Q equals $(p-1)^n - p + 1$. Such antipodes that do not lie on the linear function that passes through Q may be referred to as Q's *nonlinear antipodes*. While each antipode of Q is π radians from Q, pairs of those antipodes may be $\leq \pi$ radians from each other. Whereas consolidation of a pair of linear antipode vectors results in their annihilation, consolidation of a pair of nonlinear antipode vectors may result in their annihilation or in a vector of nonzero magnitude.

In $M^3_{G(p)}$, one may compute the Euler characteristics of polyhedra such as 3-orthotopes and their duals. (See Table 1 below.)

In $M^3_{G(p>2)}$, 3-orthotopes and their duals do not have equal Euler characteristics. In such spaces, 3-orthotopes have negative Euler characteristics whereas their duals have positive Euler characteristics > 2. Ascertaining whether such unequal characteristics indicate non-homeomorphism between 3-orthotopes and their duals or whether those characteristics indicate genera or anti-genera may warrant further exploration.

Table 1: For Computation of Euler Characteristics in $M^3_{G(p)}$

Polyhedron	**V**	**F**	**E**
3-orthotope	p^3	$3p$	$\frac{(p^3)(3(p-1))}{2}$
3-o. dual/fusil	$3p$	p^3	$3p^2$

Applications for multipolar number systems and multipolar inner product spaces over those systems may include analysis of data captured via multipolar interval scale items. Item responses and/or their scale values may be analysed by orthant for qualitative classification and, with the length of a vector defined as equal to the square root of its multipolar inner product with itself ($\|v\| = \sqrt{v \boxdot v}$), one may calculate the angle between two vectors using the equation $\angle(u, v) = \arccos \frac{u \boxdot v}{\|u\|\|v\|}$ as they might for an inner product space proper. In so doing one may have the ability to compute cosine similarity analysis in multipolar inner product spaces.

References

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